Samantha Parr
Physics 6th period

MY SUB-LIME LAB: "LIME OVER YOUR HEAD"

In this lab, I swung a mass (AKA a lime) at the end of a string in the vertical plane and tried to predict and measure its lowest possible speed when the string tension at the top of rotation was zero. Using this predicted velocity value, I predicted and measured the period of rotation, which is how many seconds it takes for the object to complete a rotation. I compared the predicted and measured periods to assess the accuracy of my experiment, considering all imperfections and causes of error. Because the velocity of the object is not constant throughout its rotation, I had to consider the separate velocities at the top and bottom points of rotation using principles of the conservation of energy.

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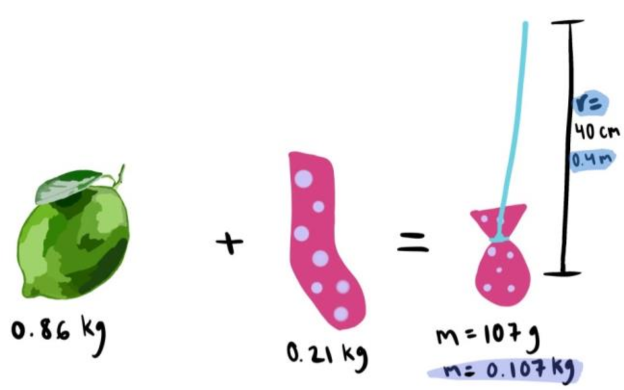
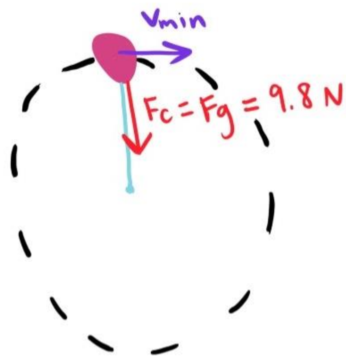
Experimental Design

In this experiment, I swung an object at the end of a string in the vertical plane at a constant velocity where there would be no string tension at the top of the object's rotation. For my "object," I put an 86-gram lime in a sock and tied it to the end of a shoelace. Altogether, the mass of the lime and the sock amounted to 107 grams, or 0.107 kilograms. I tied the shoelace so that the radius of curvature (the length from the center of the lime to my hand) was 40 cm, or 0.4 meters.



Identify the forces

We must note that at the top of the lime's revolution, there is no string tension. Tension is a force, and it points towards the center of curvature at all other points in the revolution. The force of gravity (Fg) is also present at all times, and factors into the net force value (AKA centripetal force, or Fc). Fc is the sum of all forces acting on the object at any one instant. When there is no tension at the top, however, the only force acting on the lime is the force of gravity. Thus, Fc = Fg. Essentially, the lime is now an object falling solely due to gravity.



$$F_c = F_g = \frac{mv^2}{r}$$

$$\cancel{m}g = \frac{\cancel{m}v^2}{r}$$

$$g = \frac{v^2}{r}$$

$$0.4 \times 9.8 = \frac{v^2}{\cancel{0.4}}$$

$$\sqrt{3.92} = \sqrt{v^2}$$

$$\boxed{1.98\ \text{m/s} = v}$$

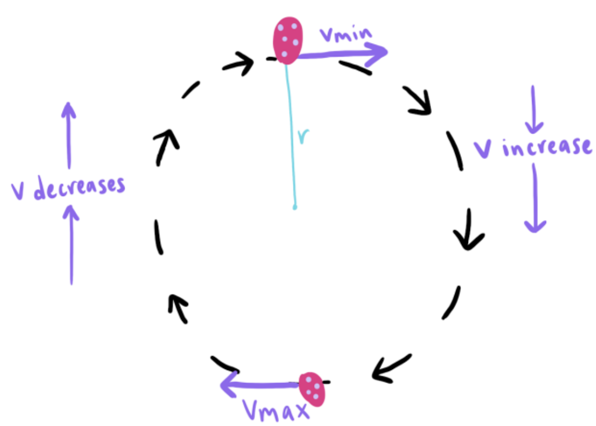
predicted Vmin (top)

Determine the velocity of the lime at the top of revolution

Assuming that the only force acting on the lime is gravity, we can set the equation Fc = (mv^2)/r equal to Fg. This is the equation for the magnitude of an object's centripetal force, which is the force that acts on an object moving in a circular path. Fc is perpendicular to the tangential velocity of the object as long as it is moving in a circular path. Substituting in 9.8 m/s^2 for the acceleration due to gravity (g) and 0.4 m for the radius of curvature 'r,' we get that the predicted velocity of the lime at the top of its orbit is 1.98 m/s.

Use principles of conservation of energy to incorporate speed increase of falling mass in computing the average velocity

The textbook problem that this lab is modeled after asks students to assume that the velocity of the rotating object is constant. In real life, this is not true. After my lime hits its peak position at the top of its orbit, it falls through space until the string catches it at the bottom of its orbit. As the lime falls, it increases in velocity. It gains vertical acceleration. Then, at the bottom of its orbit, the lime cannot gain any more vertical acceleration and only moves in the horizontal direction for a small fraction of time. Then, the lime climbs back up to the top of its orbit. As it climbs, its velocity slowly decreases to a brief stop at the top once again. Thus, the velocity of an object revolving around a center point on a string is not constant.



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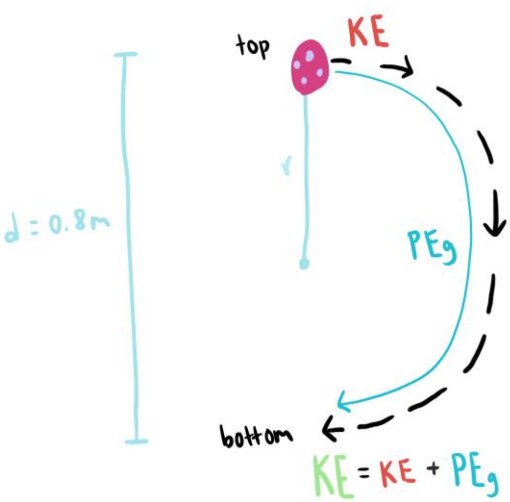
Find velocity at bottom of revolution

Considering this, we can use principles of the conservation of energy to incorporate the lime's changing velocity at the top and bottom of its orbit in computing its average velocity. The definition of the conservation of energy states that energy cannot be created or destroyed, but can rather be transformed from one form to another. So, the energy of the rotating lime is not lost or gained, but is rather represented in different amounts of kinetic or potential energy at different points of rotation. First, we can find the amount of kinetic energy at the top of orbit. Subbing in 0.107 kg for mass (m) and 1.98 m/s for velocity (v), which we determined earlier, we get that the lime possesses 0.21 Joules of energy at the top of its orbit. Next, we can calculate the energy that the lime gains as it falls. This is represented by PEg, which is the change in gravitational potential energy, or m*g*h. H is the total height/distance covered by the lime as it falls from the top to the bottom of its rotation. Since the radius of curvature is 0.4 m, the circle's diameter would be 0.8 meters. This is 'h.' We get that the lime experiences a change in gravitational potential energy of 0.84 Joules when falling from top to bottom. To find the total amount of kinetic energy that the lime holds, we can add 0.21 and 0.84 and set that equal to 1/2mv^2 to solve for v. This time, this new velocity (v) represents the maximum velocity of the lime, at the bottom of its orbit. This max velocity equals 4.43 m/s.



① Find KE at top

$$KE_{min} = \tfrac{1}{2}mv^2$$
$$= \tfrac{1}{2}(0.107)(1.98^2)$$
$$KE_{min} = 0.21\text{ J}$$

② Find PE_g at bottom

$$PE_g = m \cdot g \cdot h$$
$$= 0.107 \cdot 9.8 \cdot 0.8$$
$$PE_g = 0.84\text{ J}$$

③ Find V_{max}

$$KE_{max} = \frac{1}{2}mv^2$$
$$KE_{min} + PE_g =$$
$$0.84 + 0.21 = \tfrac{1}{2}mv^2$$
$$1.0486 = \tfrac{1}{2}(0.107)v$$
$$\frac{1.046}{0.0535} = \frac{0.0535v^2}{0.0535}$$
$$\sqrt{19.6} = \sqrt{v^2}$$
$$4.43\text{ m/s} = V_{max}$$

Use average velocity to predict average period

Finally, we need to average the minimum and maximum velocities in order to find the predicted period of rotation (when considering speed changes). We get that the average velocity is 3.2 m/s. We'll multiply the inverse of 3.2 m/s by [2pi(r) m/1 rev], which represents the distance that the lime travels in one revolution, or the circumference. We find that the period of rotation for the lime when there is no tension at the top, considering speed changes and the conservation of energy, should be 0.79 seconds per revolution.

④ Average V_{min} + V_{max}

$V_{min} = 1.98$ m/s
$V_{max} = 4.43$ m/s

$$\frac{1.98 + 4.43}{2} = \boxed{3.2\text{ m/s} = V_p}$$

$$\frac{3.2\text{ m}}{1\text{ s}} \rightarrow \frac{1\text{ s}}{3.2\ \cancel{m}} \times \frac{2\pi r\ \cancel{m}}{1\text{ rev}}$$
$$= \frac{2\pi(0.4)\text{ s}}{3.2\text{ rev}}$$
$$= \frac{2.513\text{ s}}{3.2\text{ rev}}$$
$$T_p = 0.79\text{ s/rev}$$

period if acknowledge speed changes

Experiment: find measured values of period and velocity

In order to find the measured value of the period, I recorded myself swinging the lime in front of me in the vertical plane. I found that it took the lime 4.5 seconds to complete 5 revolutions. Since period is [seconds/revolutions], I set up the fraction as 4.5 sec/5 revs. Simple division resulted in a measured period of 0.9 s/rev.

	start	end
Time elapsed (s)	0	4.5
# of revolutions	0	5

$$\text{period} = \frac{\text{sec}}{\text{revs}} = \frac{4.5\ \text{s}}{5\ \text{revs}} = \underset{\text{measured period}}{0.9\ \text{s/rev} = T_m}$$

To find the measured velocity, I took my period of 0.9 s/rev and multiplied it by [1 rev/2(pi)r m], which is the distance for one revolution. Since this gets you a fraction in s/m and I need m/s for velocity, I switched the numerator and denominator. I got an average measured velocity of 2.79 m/s.

$$\frac{0.9\ \text{s}}{\cancel{1\ \text{rev}}} \cdot \frac{\cancel{1\ \text{rev}}}{2\pi r\ \text{m}} = \frac{0.9\ \text{s}}{2\pi(0.4)\ \text{m}}$$

$$= \frac{0.9\ \text{s}}{2.51\ \text{m}} \rightarrow \frac{2.51\ \text{m}}{0.9\ \text{s}} = \boxed{2.79\ \text{m/s} = V_m}$$

Compare measured and predicted values for period and velocity

In order to measure the accuracy of my measured experiment compared to my hypothetical calculations for the value of the period and velocity, I found the percent error between the two. The smaller your percent error is, the closer your measured value is to your predicted value and the more accurate your experiment is. By comparing my measured period (0.9 s/rev) to my predicted period (0.79 s/rev), I found a 13.9% error. By comparing my measured velocity (2.79 m/s) to my predicted velocity (3.2 m/s), I found a 12.8% error. Both percent errors are low, implying that my experiment was accurate in determining the period and average velocity of the lime swinging in a circle in the vertical plane.

period

$T_m = 0.9$ s/rev

$T_p = 0.79$ s/rev

$$\%\ \text{error} = \left|\frac{T_m - T_p}{T_p}\right| = \left|\frac{0.9 - 0.79}{0.79}\right| = 0.139 \times 100\% = 13.9\%\ \text{error}$$

velocity

$V_m = 2.79$ m/s

$V_p = 3.2$ m/s

$$\%\ \text{error} = \left|\frac{2.79 - 3.2}{3.2}\right| = 0.128 \times 100\% = 12.8\%\ \text{error}$$

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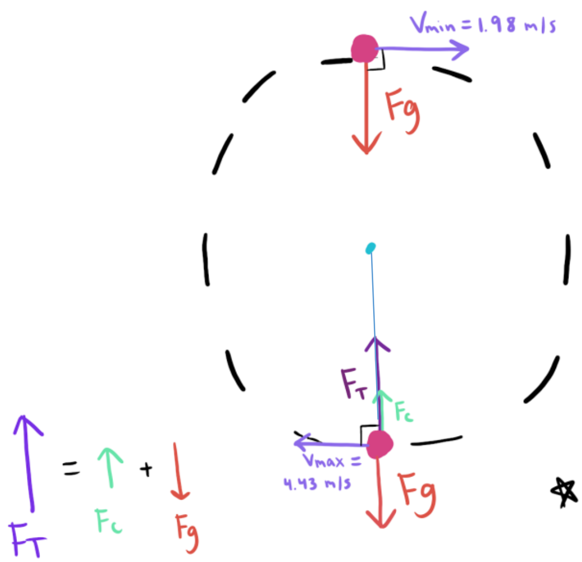
Calculate force of tension at bottom



$$F_c = \frac{mv^2}{r} = \frac{(0.107)(4.43^2)}{0.4}$$

$$\boxed{F_c = 5.25\text{ N}}$$

$$F_g = mg = (0.107)(9.8)$$

$$\boxed{F_g = 1.05\text{ N}}$$

$$F_T = F_c + F_g = 5.25 + 1.05$$

$$\boxed{F_T = 6.3\text{ N}}$$

At the bottom of the lime's revolution, there is tension and the force of gravity acting on it. Fc, the centripetal force, is the sum of Ft (tension) and Fg. Since Fg is negative, we can rearrange the equation to say Ft = Fc + Fg. We'll find the magnitude of Fc using the centripetal force equation. This is 5.25 N. Fg is just mass times acceleration due to gravity (9.8 m/s^2), which is 1.05 N. Then, we'll sub in those two values into our equation for the force of tension, adding them up to get that tension equals 6.3 N. At the bottom of the revolution, when the lime's velocity is the highest, there are 6.3 Newtons of force in the string.

Conclusion

In this lab, my objective was to compare theoretical calculations for a rotating object's period and velocity (using centripetal force and conservation of energy) to measured values produced from my own real-life experiment. In my experiment, I put a lime in a sock and tied it to a shoelace. I swung it around five times in a circle in the vertical plane, and used a timer to track how long that took. This is how I found my measured period and velocity.

I got a 13.9% error between my measured and predicted periods, and a 12.8% error between my measured and predicted average velocities. Because these are both low percent errors, we can conclude that my experiment was successful. However, there could be ways to minimize this error even further. For example, one source of error was likely from my hand. It's nearly impossible to keep your wrist completely still while the object rotates around, so I probably added an extra source of force. Although I factored my hand's movement into the radius, I still likely moved it more than I accounted for. My extra hand movement could be the reason why my measured velocity is less than my predicted velocity, as I could have subconsciously been influencing the lime's movement and caused it to move too slowly. This also affects the period, as they deal with the same values for distance and time. Because period is seconds per revolution, the fact that my measured velocity was too slow made my period too high (it took a longer amount of time to make one compete revolution than was predicted). Next time, I would take extra care to keep my hand as still as possible to produce the most accurate results.

Overall, my experiment was successful in measuring the velocity and period of a lime spinning in circles in the vertical plane. By grappling with the different numbers and calculations and trying to make sense of them in a real-life context, I feel that this lab helped me gain a better understanding of centripetal force and rotational motion. Plus, I liked the idea of swinging a lime in circles (even though your parents tell you not to play with food!)